

Parameter Scaling in Vertical Feedback Seismomenters

When analyzing feedback seismometer performance in the Excel workbook 'Loop', the assumption is made that the Forcing Coil, the Position Sensor, the Spring and the Center of Mass are all located at a single point along the boom. This is not generally true and for the spreadsheet, the measured parameter values must be corrected so that they will appear to be acting at the assumed point. In the spreadsheet, these corrected values are characterized as 'effective' values.

Additionally, the workbook incorrectly computes the boom's moment of inertia about its pivot, by assuming that its mass is concentrated at a single point located at the center of mass. The boom is in fact a distributed structure, having a moment of inertia about its pivot which is greater than that of the point mass. In order to have everything acting at a single point, two mass values must be used. The first is the actual mass, acting at the center of mass, corrected to locate it at the common reference point, and used in certain of the calculations. The second is a larger mass which results in the proper rotational moment of inertia when located at the common point and which is used in other calculations as appropriate. Currently there is no mechanism for including this second mass value in the spread sheet, though that is in the process of being corrected.

Referring to Figures 1 and 2, the variables which we will use are related as follows:

- 1) $M_0 = M d_1/d$ kg
- 2) $M_1 = M (d_3/d)^2$ kg
- 3) $r_t = r d/d_4$ V/m
- 4) $G_n = G d_2/d$ N/A
- 5) $K_0 = K(d_5/d)^2$ N/m

(Note that the spreadsheet derives K_0 from the measured spring-mass period and it is not required to be entered as an input parameter.) Taking K_0 from the spreadsheet, we can compute the 'measured' spring constant of the real spring, located at distance d_5 as

- 6) $K = K_0(d/d_5)^2$ N/m

Here, M_0 , M_1 , r_t , G_n and K_0 are the 'effective' values used in the spreadsheet, and M , r , G and K are the 'true' values, as would be measured at the actual locations of the various elements. Note that the value of 'd' is arbitrary and any convenient value may be used. It would probably be simplest to make $d = d_1$, the distance to the center of mass, or perhaps $d = d_2$ the distance to the forcing coil axis, just so long as you use the same value for 'd' in all calculations.

M is the measured mass of the boom assembly, located at the center of mass, which is at horizontal distance d_1 from the pivot.

r is the sensitivity of the position sensor (with its amplifiers) as measured at its center, located at radial distance d_4 from the pivot.

K is the incremental force constant of the spring, located at a radial distance d_5 from the pivot.

G is the force constant of the feedback coil, located at a radial distance d_2 from the pivot.

Distances d_2 , d_4 and d_5 are physical dimensions which may be measured. The location of the center of mass, d_1 and the radius of inertia, d_3 will require some experimentation to determine. d_1 may be determined by hanging the boom assembly from a convenient point and estimating where a vertical line through the support would intersect the plane of symmetry of the boom, taking d_1 as the distance from the pivot to the intersection point.

To determine d_3 requires somewhat more work. One accurate method is to incorporate the boom into a torsion pendulum and measure its period. A suitable length of music wire can be used for the torsion spring, suspending the boom on its side from a point chosen so that the hinge axis is exactly vertical. After the oscillation period of the boom, T_b is measured, the boom is replaced by a circular disk of known mass supported horizontally from its center, having mass M_d and radius R_d , and its period of oscillation T_d is measured. Knowing that the period of oscillation of a torsion pendulum, in general, is given by:

$$7) \quad T = 2\pi\sqrt{I/K} \text{ sec}$$

where I is the moment of inertia of the suspended object and K is the torsion constant of the suspension wire, it can be determined that the moment of inertia of the boom about its center of mass, when suspended from the same wire as the disk is,

$$8) \quad I_{bc} = (M_d R_d^2 / 2) (T_b / T_d)^2$$

Now, having determined the values of I_{bc} and d_1 , we can weigh the boom to determine its mass, M , and then make use of the parallel axis theorem to solve for the moment of inertia of the boom about its pivot axis,

$$9) \quad I_{bp} = M d_1^2 + I_{bc}$$

This makes the radius of inertia of the boom about its pivot,

$$10) \quad d_3 = \sqrt{(d_1^2 + I_{bc}/M)}$$

which is the value we were seeking.

$$\text{Then from 2) we have that} \quad M_1 = M (d_3/d)^2 = M (d_1^2 + I_{bc}/M) / d^2$$

which allows us to calculate M_1 as a function of the rotational moment of inertia which was measured.

$$11) \quad M_1 = M d_1^2 / d^2 + I_{bc} / d^2$$

Real World

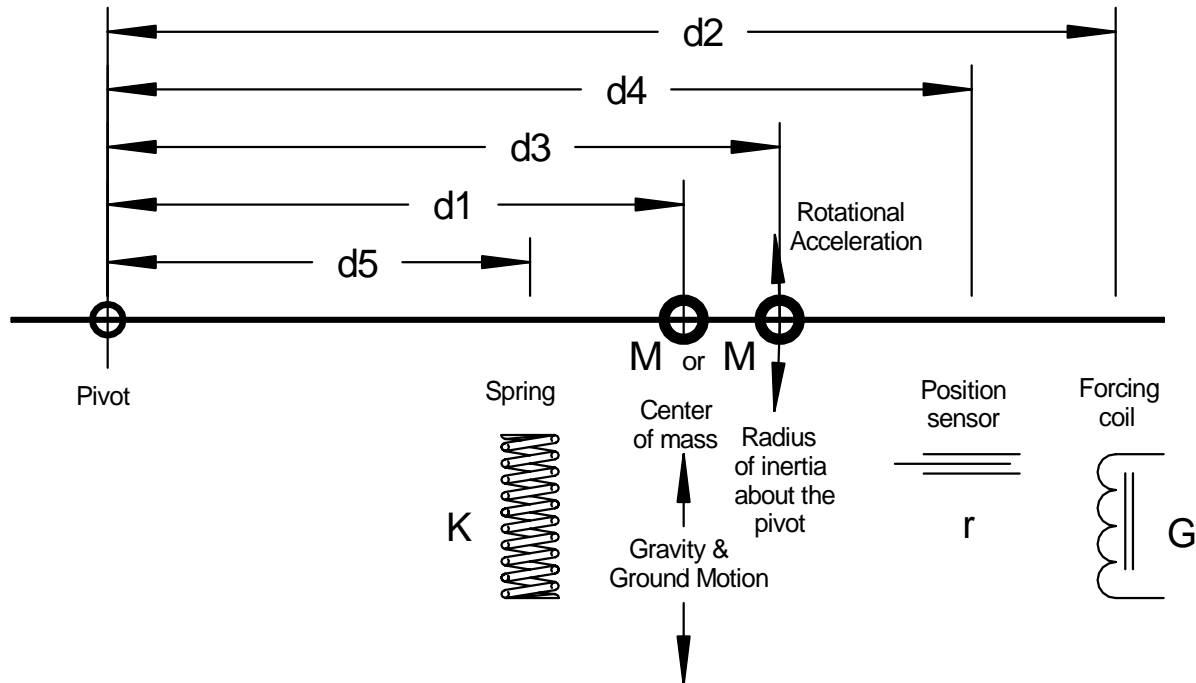


Figure 1

Point-Mass Model

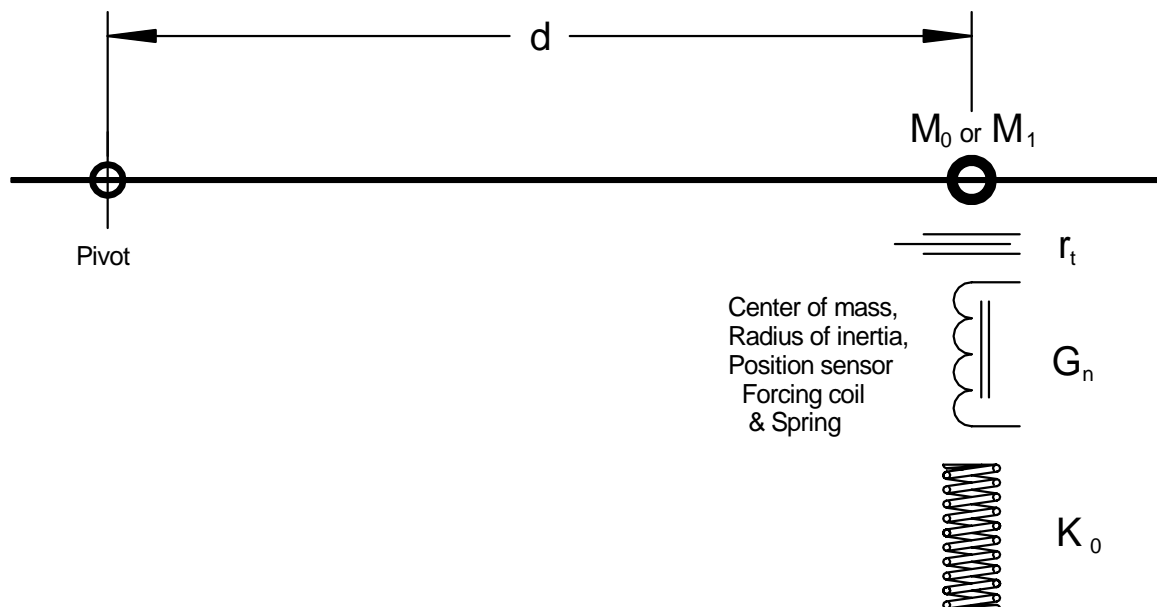


Figure 2

Conversions between linear and rotary motion of the boom.

These all assume that the boom rotation angle θ , is sufficiently small that the approximation, $\sin\theta = \theta$ radians, is valid. Here 'd' is the distance from the pivot to the point of measurement.

- 12) $dz = d \cdot d\theta$ Rotation $d\theta$ to vertical displacement dz
- 13) $V = dz/dt = r d\theta/dt = d \cdot \omega_p$ Rotational rate ω_p to vertical velocity V
- 14) $A = d^2z/dt^2 = r d^2\theta/dt^2 = d \cdot a$ Rotational acceleration a to vertical acceleration A
- 15) $d\theta = dz/d$ Vertical displacement d to rotation $d\theta$
- 16) $\omega_{bp} = d\theta/dt = 1/r dz/dt = V/d$ Vertical velocity d to rotational rate ω_{bp}
- 17) $a = d^2\theta/dt^2 = 1/r d^2z/dt^2 = A/d$ Vertical acceleration A to rotational acceleration a

Note that ω_{bp} is the instantaneous rate of rotation of the boom about its pivot (in radians per second), as distinct from plain ω , the frequency of an assumed sinusoidal ground motion, also expressed in radians per second.

Equations of Motion

Ground motion acting on the boom

Given that 'F' is the acceleration force on the center of mass due to ground motion.

- 18) $F = M A_g$, where A_g is the acceleration associated with ground motion.

Torque, 'Q', acting on the boom assembly due to ground motion

- 19) $Q = F d_1 = M d_1 A_g = M_0 d A_g$

Rotational Acceleration, 'a', of the boom, due to ground motion

- 20) $a = Q / I_{bp} = M_0 d A_g / I_{bp} = A_g d M_0 / (M_1 d^2) = A_g/d M_0/M_1$

Linear acceleration of the point at distance 'd' from the pivot

- 21) $A_d = d a = A_g M_0/M_1 = A_g d d_1/d_3^2 = A_g d/(d_1+I_{bc}/M d_1)$

So the acceleration of point 'd' is decreased by the factor M_0/M_1 from the ground acceleration, where M_0/M_1 also $= d d_1/d_3^2 = d/(d_1+I_{bc}/M d_1)$

Note that in the STS-2 block diagram, in which 'd' is chosen $= d_2$, (which they also assume $= d_4$), they show this factor as $d_1 d_2/d_3^2$ which agrees with 21) when 'd' is replaced by 'd₂'.

Feedback coil force acting on the boom

Assume that the feedback coil exerts a force, 'F', on the boom at distance 'd₂' from the pivot. The torque 'Q' applied by the feedback coil to the boom is

$$22) \quad Q = F d_2 \text{ where } F \text{ is the feedback force.}$$

The angular acceleration, a, due to the torque 'Q'

$$23) \quad a = Q/I_{bp} = F d_2/I_{bp}$$

From 14) this results in a vertical acceleration, 'A', at radius 'd', of

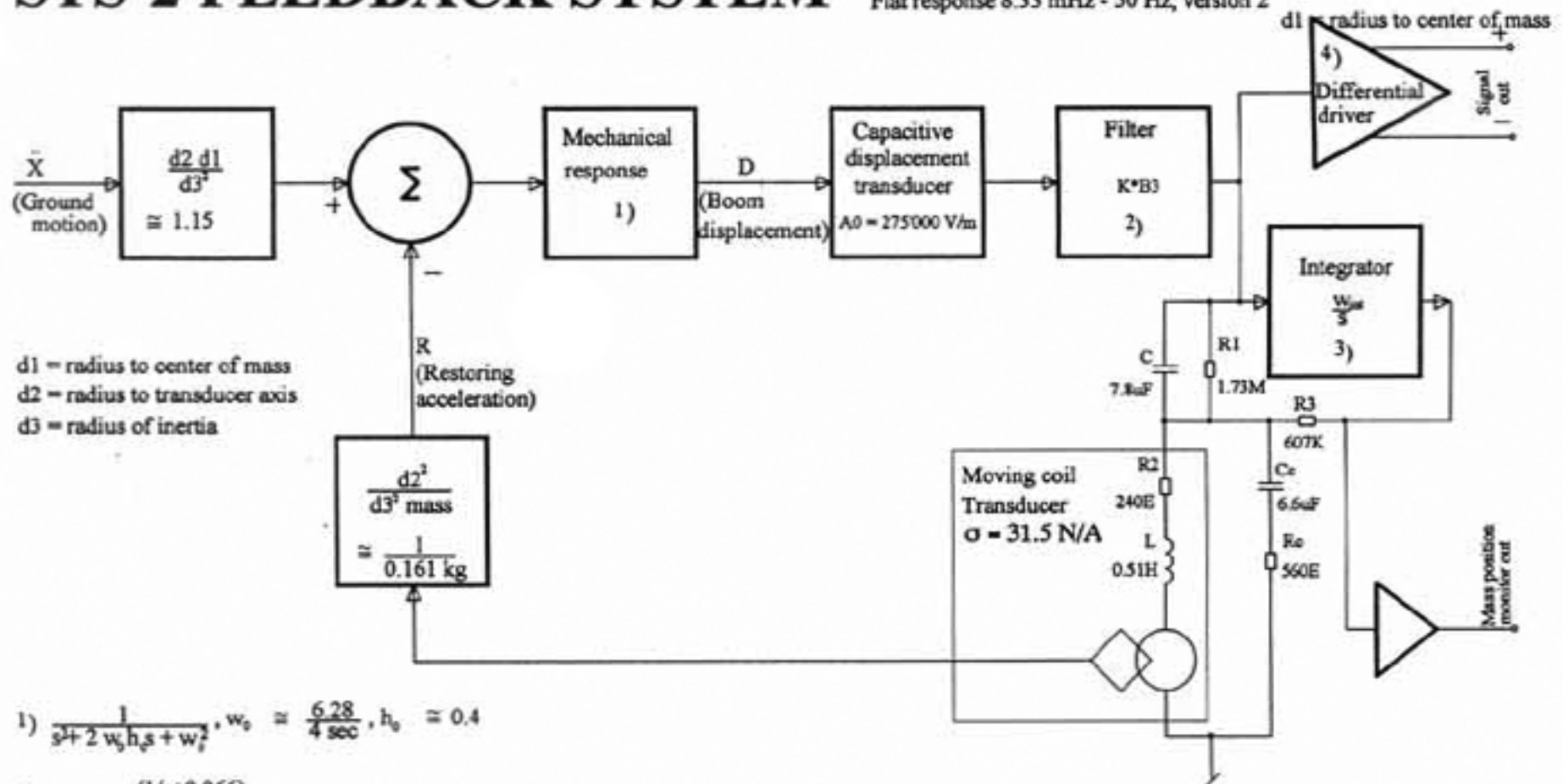
$$24) \quad A = a d = F/M_1 d_2/d = F d d_2 / M d_3^2$$

In the STS-2 block diagram it is assumed that $d = d_2$ so the acceleration 'A' of the point at distance 'd₂' due to a feedback coil force of 'F'

$$25) \quad A = F d_2^2 / M d_3^2 \text{ which agrees with the STS-2 block diagram.}$$

STS-2 FEEDBACK SYSTEM

Flat response 8.33 mHz - 50 Hz, version 2



$$1) \frac{1}{s^2 + 2\omega_0 h_0 s + \omega_0^2}, \omega_0 \approx \frac{6.28}{4 \text{ sec}}, h_0 \approx 0.4$$

$$2) K = \frac{(1/s + 0.066)}{(1/(475s) + 0.066)} \quad (\text{Inverse filter})$$

$$B_3 = \frac{1}{(1 + 7.52e-5s)(1 + 9.94e-5s + 4.72e-9s^2)} \quad (\text{Bessel 3rd order 1600 Hz})$$

$$3) w_{int} = \frac{1}{77.1 \text{ sec}}$$

$$4) \text{ With low-pass filter: } \frac{1}{s/w_{int} + 1}, w_{int} = 6.28 \cdot 40 / \text{sec}$$